

Revisiting the simplified thermo-hydraulics module THM: in
DONJON5 code

A. HÉBERT

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1 Thermal conduction equation in the fuel and clad

The temperature distribution in the fuel rod is given by the Fourier's law of heat conduction, written as

$$\nabla \cdot [k(T)\nabla T(\mathbf{r}, t)] + Q(\mathbf{r}, t) = \frac{\partial}{\partial t} [\rho(T)C(T)T(\mathbf{r}, t)] \quad (1.1)$$

where

$T(\mathbf{r}, t)$ = temperature distribution in the fuel rod, including the clad (K)

$Q(\mathbf{r}, t)$ = fission power distribution in fuel (W/m³)

$k(T)$ = thermal conductivity of the fuel and clad (W/m/K)

$\rho(T)$ = density of the fuel and clad (kg/m³)

$C(T)$ = specific heat capacity of the fuel and clad (J/K/kg).

We consider that the conduction process in the fuel rod has a *radial symmetry*, so that the temperature is only a distribution of the radial coordinate r . Equation (1.1) can be rewritten as

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r k(T) \frac{\partial}{\partial r} T(r, t) \right] + Q(r, t) = \frac{\partial}{\partial t} [\rho(T)C(T)T(r, t)]. \quad (1.2)$$

The THM: module is based on a *constant surface* discretization of the fuel rod. A change of variable is performed on r , as

$$A(r) = \frac{r^2}{2} \Rightarrow dA = r dr \quad (1.3)$$

so that Eq. (1.2) can be rewritten as

$$2 \frac{\partial}{\partial A} \left[A k(T) \frac{\partial}{\partial A} T(A, t) \right] + Q(A, t) = \frac{\partial}{\partial t} [\rho(T)C(T)T(A, t)]. \quad (1.4)$$

Equation (1.4) is discretized in A using the *mesh centered finite difference* (MCFD) method, based on the low order difference relations

$$\left. \frac{\partial T}{\partial A} \right|_{i-1/2-} \simeq 2 \frac{T_{i-1/2} - T_{i-1}}{\Delta A_{i-1}} \quad \text{and} \quad \left. \frac{\partial T}{\partial A} \right|_{i-1/2+} \simeq 2 \frac{T_i - T_{i-1/2}}{\Delta A_i} \quad (1.5)$$

where we have set $\Delta A_i = A_{i+1/2} - A_{i-1/2}$. We define I equal-size volumes in the fuel, a gap, and I_c equal-size volumes in the clad. Fuel rod interfaces are located at $A_{i+1/2}$ with $1 \leq i \leq I$ and $A_{1/2} = 0$, as depicted in Fig. 1.

The *heat flux* $\Phi(r)$, expressed in W/m², is defined as

$$\Phi(r) = -k(T) \frac{\partial}{\partial r} T(r, t) = -r k(T) \frac{\partial}{\partial A} T(A, t). \quad (1.6)$$

We state the heat flux continuity condition at each point $A_{i-1/2}$ as $\Phi(r_{i-1/2-}) = \Phi(r_{i-1/2+})$. This condition is written

$$k_{i-1} \left. \frac{\partial T}{\partial A} \right|_{i-1/2-} = k_i \left. \frac{\partial T}{\partial A} \right|_{i-1/2+}. \quad (1.7)$$

The discretization process produces a symmetric, positive-definite and tri-diagonal linear system with the introduction of the following time-dependent mesh-centered finite difference coefficients of unit W/m/K:

$$D_{i+1/2} = \frac{4A_{i+1/2}}{\frac{\Delta A_i}{k_i} + \frac{\Delta A_{i+1}}{k_{i+1}}} = \frac{4A_{i+1/2}}{\Delta A_i + \Delta A_{i+1}} \left[\frac{1}{T_i - T_{i+1}} \int_{T_{i+1}}^{T_i} dT k(T) \right] \quad (1.8)$$

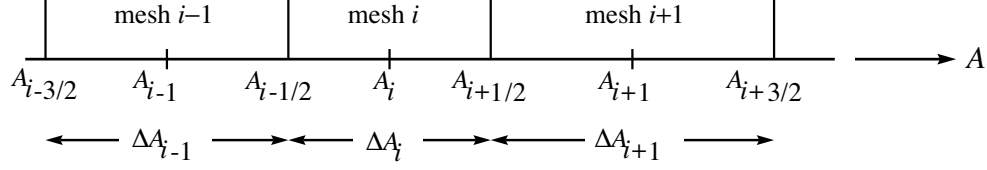


Figure 1: Definition of the regions in 1D cylindrical geometry.

$$E_{i+1/2} = \frac{4A_{i+1/2} k_i}{\Delta A_i} = \frac{4A_{i+1/2}}{\Delta A_i} \left[\frac{1}{T_i - T_{i+1/2}} \int_{T_{i+1/2}}^{T_i} dT k(T) \right] \quad (1.9)$$

$$F_{i+1/2} = \frac{4A_{i+1/2} k_{i+1}}{\Delta A_{i+1}} = \frac{4A_{i+1/2}}{\Delta A_{i+1}} \left[\frac{1}{T_{i+1/2} - T_{i+1}} \int_{T_{i+1}}^{T_{i+1/2}} dT k(T) \right] \quad (1.10)$$

and

$$G_i = \mathcal{H}_{\text{gap}} r_i \quad (1.11)$$

where \mathcal{H}_{gap} is the heat exchange coefficient of the gap, expressed in W/m²/K. Equations (1.8) to (1.10) are based on a quadrature of the thermal conductivity with a linear variation of the temperature. This procedure permits to obtain an accurate temperature distribution in the fuel with a limited number I of equal-size volumes.

Combining Eqs. (1.5) and (1.7) we obtain an expression on the interface temperatures as

$$T_{i-1/2} = \frac{\Delta A_i k_{i-1} T_{i-1} + \Delta A_{i-1} k_i T_i}{\Delta A_i k_{i-1} + \Delta A_{i-1} k_i} \quad (1.12)$$

and

$$T_{i+1/2} = \frac{\Delta A_{i+1} k_i T_i + \Delta A_i k_{i+1} T_{i+1}}{\Delta A_{i+1} k_i + \Delta A_i k_{i+1}}. \quad (1.13)$$

After substitution of Eq. (1.12) into Eqs. (1.5), we obtain our first mesh-centered finite difference relation as

$$\left. \frac{\partial T}{\partial A} \right|_{i-1/2+} = 2k_{i-1} \frac{T_i - T_{i-1}}{\Delta A_i k_{i-1} + \Delta A_{i-1} k_i}. \quad (1.14)$$

Using a similar approach, we obtain a second mesh-centered finite-difference relation as

$$\left. \frac{\partial T}{\partial A} \right|_{i+1/2-} = 2k_{i+1} \frac{T_{i+1} - T_i}{\Delta A_{i+1} k_i + \Delta A_i k_{i+1}}. \quad (1.15)$$

Setting $1 \leq i < I$ in Eq. (1.4) as the fuel domain and taking the derivative of heat flux term in interval $A_{i-1/2} < A < A_{i+1/2}$, we obtain

$$\begin{aligned} & \frac{2}{\Delta A_i} \left[A_{i+1/2} k_i(t) \left. \frac{\partial T(t)}{\partial A} \right|_{i+1/2-} - A_{i-1/2} k_i(t) \left. \frac{\partial T(t)}{\partial A} \right|_{i-1/2+} \right] + Q_i(t) \\ &= \frac{\partial}{\partial t} [\rho_i(t) C_i(t) T_i(t)]. \end{aligned} \quad (1.16)$$

Substituting Eqs. (1.14) and (1.15) in Eq. (1.16), we obtain the finite difference relation for the temperature unknowns inside the fuel rod:

$$\begin{aligned} & 4 \left[A_{i+1/2} \frac{T_{i+1}(t) - T_i(t)}{\frac{\Delta A_i}{k_i(t)} + \frac{\Delta A_{i+1}}{k_{i+1}(t)}} - A_{i-1/2} \frac{T_i(t) - T_{i-1}(t)}{\frac{\Delta A_{i-1}}{k_{i-1}(t)} + \frac{\Delta A_i}{k_i(t)}} \right] \\ &+ \Delta A_i Q_i(t) = \Delta A_i \frac{\partial}{\partial t} [\rho_i(t) C_i(t) T_i(t)]. \end{aligned} \quad (1.17)$$

The corresponding tri-diagonal relations in temperature for internal points are

$$\begin{aligned} & \Delta A_i \frac{\partial}{\partial t} [\rho_i(t) C_i(t) T_i(t)] - D_{i-1/2}(t) T_{i-1}(t) \\ & + [D_{i-1/2}(t) + D_{i+1/2}(t)] T_i(t) - D_{i+1/2}(t) T_{i+1}(t) = \Delta A_i Q_i(t). \end{aligned} \quad (1.18)$$

Setting $i = I$ and substituting Eqs. (1.5) and (1.15) in Eq. (1.16), we obtain the finite difference relation between the temperatures inside the outer fuel ring (T_I) and at the surface of the fuel pin ($T_{I+1/2}$):

$$\begin{aligned} & 4 \left[\frac{A_{I+1/2} k_I(t)}{\Delta A_I} [T_{I+1/2}(t) - T_I(t)] - A_{I-1/2} \frac{T_I(t) - T_{I-1}(t)}{\frac{\Delta A_{I-1}}{k_{I-1}(t)} + \frac{\Delta A_I}{k_I(t)}} \right] \\ & + \Delta A_I Q_I(t) = \Delta A_I \frac{\partial}{\partial t} [\rho_I(t) C_I(t) T_I(t)]. \end{aligned} \quad (1.19)$$

The corresponding tri-diagonal relation for fuel surface temperature is

$$\begin{aligned} & \Delta A_I \frac{\partial}{\partial t} [\rho_I(t) C_I(t) T_I(t)] - D_{I-1/2}(t) T_{I-1}(t) \\ & + [D_{I-1/2}(t) + E_{I+1/2}(t)] T_I(t) - E_{I+1/2}(t) T_{I+1/2}(t) = \Delta A_I Q_I(t). \end{aligned} \quad (1.20)$$

The *h-gap relations* link the temperature gradients at the surface of the fuel pin and at the inner clad surface. The heat fluxes at the fuel surface ($r_{I+1/2}$), inside the gap at r_{I+1} and at the inner clad surface ($r_{I+3/2}$) are related through

$$r_{I+1/2} \Phi(r_{I+1/2-}) = r_{I+1} \Phi(r_{I+1}) = r_{I+3/2} \Phi(r_{I+3/2+}). \quad (1.21)$$

Equation (1.21) is used together with the *h-gap correlation*, written as

$$\Phi(r_{I+1}) = -\mathcal{H}_{\text{gap}} \Delta T = \mathcal{H}_{\text{gap}} [T_{I+1/2}(t) - T_{I+3/2}(t)] \quad (1.22)$$

so that the left equality in Eq. (1.21) is written

$$-r_{I+1/2} k_I \left. \frac{\partial T(t)}{\partial A} \right|_{I+1/2-} = \mathcal{H}_{\text{gap}} \frac{r_{I+1}}{r_{I+1/2}} [T_{I+1/2}(t) - T_{I+3/2}(t)] \quad (1.23)$$

and the right equality in Eq. (1.21) is written

$$\mathcal{H}_{\text{gap}} \frac{r_{I+1}}{r_{I+3/2}} [T_{I+1/2}(t) - T_{I+3/2}(t)] = -r_{I+3/2} k_{I+2} \left. \frac{\partial T(t)}{\partial A} \right|_{I+3/2+}. \quad (1.24)$$

The position of mesh value r_{I+1} in the gap is selected to produce a constant heat flux:

$$\begin{aligned} \mathcal{H}_{\text{gap}} r_{I+1} [T_{I+1/2}(t) - T_{I+3/2}(t)] &= 2\mathcal{H}_{\text{gap}} r_{I+3/2} [T_{I+1}(t) - T_{I+3/2}(t)] \\ &= 2\mathcal{H}_{\text{gap}} r_{I+1/2} [T_{I+1/2}(t) - T_{I+1}(t)] \end{aligned} \quad (1.25)$$

so that

$$r_{I+1} = \frac{2 r_{I+1/2} r_{I+3/2}}{r_{I+1/2} + r_{I+3/2}}. \quad (1.26)$$

Using Eqs. (1.5), we obtain

$$-2r_{I+1/2} k_I \frac{T_{I+1/2}(t) - T_I(t)}{\Delta A_I} = \mathcal{H}_{\text{gap}} \frac{r_{I+1}}{r_{I+1/2}} [T_{I+1/2}(t) - T_{I+3/2}(t)] \quad (1.27)$$

and

$$\mathcal{H}_{\text{gap}} \frac{r_{I+1}}{r_{I+3/2}} [T_{I+1/2}(t) - T_{I+3/2}(t)] = -2r_{I+3/2} k_{I+2} \frac{T_{I+2}(t) - T_{I+3/2}(t)}{\Delta A_{I+2}}. \quad (1.28)$$

The corresponding tri-diagonal relations for fuel and inner clad surface temperatures are

$$-E_{I+1/2}(t)T_I(t) + [E_{I+1/2}(t) + G_{I+1}(t)]T_{I+1/2}(t) - G_{I+1}(t)T_{I+3/2}(t) = 0 \quad (1.29)$$

and

$$-G_{I+1}(t)T_{I+1/2}(t) + [G_{I+1}(t) + F_{I+3/2}(t)]T_{I+3/2}(t) - F_{I+3/2}(t)T_{I+2}(t) = 0. \quad (1.30)$$

The tri-diagonal relation for clad surface temperature is similar to Eq. (1.20). It is written

$$\begin{aligned} \Delta A_{I+I_c+1} \frac{\partial}{\partial t} [\rho_{I+I_c+1}(t)C_{I+I_c+1}(t)T_{I+I_c+1}(t)] - D_{I+I_c+1/2}(t)T_{I+I_c}(t) \\ + [D_{I+I_c+1/2}(t) + E_{I+I_c+3/2}(t)]T_{I+I_c+1}(t) = E_{I+I_c+3/2}(t)T_{\text{surf}}(t) + \Delta A_{I+I_c+1}Q_{I+I_c+1}(t). \end{aligned} \quad (1.31)$$

We obtained a tridiagonal linear system of order $I + I_c + 2$ with a source term function of the external clad surface temperature T_{surf} . This system can be written

$$\left[\frac{\partial}{\partial t} \mathbb{C}(t) + \mathbb{A}(t) \right] \mathbf{T}(t) = \mathbf{S}_1(t) + T_{\text{surf}}(t)\mathbf{S}_2(t) \quad (1.32)$$

where

$$\begin{aligned} \mathbb{C} = \text{diag}(\Delta A_1 \rho_1 C_1, \Delta A_2 \rho_2 C_2, \dots, \Delta A_I \rho_I C_I, 0, 0, \Delta A_{I+2} \rho_{I+2} C_{I+2}, \dots, \\ \Delta A_{I+I_c+1} \rho_{I+I_c+1} C_{I+I_c+1}), \end{aligned} \quad (1.33)$$

$$\mathbb{A} = \begin{pmatrix} D_{3/2} & -D_{3/2} & 0 & \dots & \dots & \dots \\ -D_{3/2} & D_{3/2} + D_{5/2} & -D_{5/2} & 0 & & \\ \vdots & & \ddots & & & \\ \vdots & -D_{I-1/2} & D_{I-1/2} + E_{I+1/2} & -E_{I+1/2} & 0 & \\ \vdots & 0 & -E_{I+1/2} & E_{I+1/2} + G_{I+1} & -G_{I+1} & 0 \\ \vdots & & 0 & -G_{I+1} & G_{I+1} + F_{I+3/2} & -F_{I+3/2} \\ \vdots & & & 0 & -F_{I+3/2} & \ddots \\ \vdots & & & & & \ddots \end{pmatrix}, \quad (1.34)$$

$$\mathbf{T} = \text{col}(T_1, T_2, \dots, T_I, T_{I+1/2}, T_{I+3/2}, T_{I+2}, T_{I+3}, \dots, T_{I+I_c+1}), \quad (1.35)$$

$$\mathbf{S}_1 = \text{col}(\Delta A_1 Q_1, \Delta A_2 Q_2, \dots, \Delta A_I Q_I, 0, 0, \Delta A_{I+2} Q_{I+2}, \dots, \Delta A_{I+I_c+1} Q_{I+I_c+1}) \quad (1.36)$$

and

$$\mathbf{S}_2 = \text{col}(0, 0, \dots, 0, E_{I+I_c+3/2}(t)) \quad (1.37)$$

where region $I + 1$ corresponds to the gap and index $I + I_c + 3/2$ refers to the external clad surface.

The time-derivative is handled using a *fully implicit temporal scheme*, based on a finite-difference relation, written as

$$\left. \frac{\partial}{\partial t} \mathbb{C}(t)\mathbf{T}(t) \right|_{t_n} = \frac{\mathbb{C}(t_n)\mathbf{T}(t_n) - \mathbb{C}(t_{n-1})\mathbf{T}(t_{n-1})}{\Delta t_n} \quad (1.38)$$

where $\mathbf{T}(t_n)$ is the temperature at time t_n and Δt_n is the time step size over which the temporal derivatives of the unknowns are assumed to be constant. A variable time step size is used during the numerical procedure for decreasing the total amount of time steps and computing time.

Substituting Eq. (1.32) into Eq. (1.38), we obtain

$$-\mathbb{A}(t_n)\mathbf{T}(t_n) + \mathbf{S}_1(t_n) + T_{\text{surf}}(t_n)\mathbf{S}_2(t_n) = \frac{\mathbb{C}(t_n)\mathbf{T}(t_n) - \mathbb{C}(t_{n-1})\mathbf{T}(t_{n-1})}{\Delta t_n} \quad (1.39)$$

Rearranging terms, our implicit scheme is finally written as

$$\mathbf{T}(t_n) = \left[\frac{\mathbb{C}(t_n)}{\Delta t_n} + \mathbb{A}(t_n) \right]^{-1} \left\{ \left[\frac{\mathbb{C}(t_{n-1})}{\Delta t_n} \right] \mathbf{T}(t_{n-1}) + [\mathbf{S}_1(t_n) + T_{\text{surf}}(t_n)\mathbf{S}_2(t_n)] \right\}. \quad (1.40)$$

The *linear power* delivered from the clad to the fluid in W/m is obtained as

$$q''_{\text{fluid}}(t) = 2\pi E_{I+I_c+3/2}(t) [T_{I+I_c+1}(t) - T_{\text{surf}}(t)] \quad (1.41)$$

where the temperature in the center of mesh $I + I_c + 1$ is a function of $T_{\text{surf}}(t)$ written as

$$T_{I+I_c+1}(t) = B_1(t) + T_{\text{surf}}(t) B_2(t). \quad (1.42)$$

Substituting Eq. (1.42) into Eq. (1.41), we obtain

$$q''_{\text{fluid}}(t) = 2\pi [Z_1(t) - T_{\text{surf}}(t) Z_2(t)] \quad (1.43)$$

where $Z_1(t) = E_{I+I_c+3/2}(t) B_1(t)$ and $Z_2(t) = E_{I+I_c+3/2}(t) [1 - B_2(t)]$.

Equation (1.43) is used in transient cases. If the flow is in steady-state conditions, the linear power q''_{fluid} around the clad can be obtained directly from the steady-state fission power distribution in the fuel as

$$q''_{\text{fluid}} = \int_{S_{\text{fuel}}} d^2r Q(\mathbf{r}) \quad (1.44)$$

where $S_{\text{fuel}} = \pi r_{\text{fuel}}^2$ is the fuel cross section area.

The fuel effective temperature T_{eff} can be expressed in term of the *Rowlands formula*, a simplified correlation written as

$$T_{\text{eff}} = \frac{5}{9} T_{\text{sfuel}} + \frac{4}{9} T_{\text{center}} \quad (1.45)$$

where T_{sfuel} is the surface-fuel temperature and T_{center} is the center-pin temperature assumed to be related to its neighbour values by a linear expression:

$$T_1 = \frac{1}{2}(T_{\text{center}} + T_{3/2}). \quad (1.46)$$

The mesh-limit temperature $T_{3/2}$ can be computed iteratively using Eq. (1.7):

$$\left[\int_{T_{3/2}}^{T_1} dT k(T) \right] (T_1 - T_{3/2}) = \left[\int_{T_2}^{T_{3/2}} dT k(T) \right] (T_{3/2} - T_2) \quad (1.47)$$

and T_{center} can be obtained from Eq. (1.46) using the converged value of $T_{3/2}$.

2 Thermal convection equation in the coolant

The heat exchange relation between clad and fluid for different *heat transfer regimes* \mathcal{K} is a function of the clad surface temperature $T_{\text{surf}}(t)$ and of the saturation temperature $T_{\text{sat}}(t)$ of the coolant. If the clad surface temperature is smaller than the saturation temperature of the coolant, the heat exchange is the *single-phase* free convection regime ($\mathcal{K} = 0$) and is described by the Dittus-Boelter correlation. In the *subcooled boiling regime* ($\mathcal{K} = 1$), the bulk coolant is still subcooled ($T_{\text{fluid}}(t) < T_{\text{sat}}(t)$), while the liquid and vapor in contact with the wall are slightly above saturation. Finally, in the *saturated nucleate boiling regime* ($\mathcal{K} = 2$), nucleate boiling occurs in the fluid and its thermodynamic quality $x_{\text{th}}(t)$ is above zero. An additional component $\mathcal{H}_{\text{nb}}(t)$ must be added to the Dittus-Boelter thermal exchange coefficient in cases where $\mathcal{K} \geq 1$. The Newton's law of cooling is written

$$\frac{q''_{\text{fluid}}(t)}{2\pi r_{\text{clad}}} = \begin{cases} \mathcal{H}_c(t) [T_{\text{surf}}(t) - T_{\text{fluid}}(t)] & \text{if } \mathcal{K} = 0; \\ \mathcal{H}_{\text{nb}}(t) [T_{\text{surf}}(t) - T_{\text{sat}}(t)] + \mathcal{H}_c(t) [T_{\text{surf}}(t) - T_{\text{fluid}}(t)] & \text{if } \mathcal{K} = 1; \\ [\mathcal{H}_{\text{nb}}(t) + \mathcal{H}_c(t)] [T_{\text{surf}}(t) - T_{\text{sat}}(t)] & \text{if } \mathcal{K} = 2; \end{cases} \quad (2.1)$$

where

$\mathcal{H}_c(t)$ = Dittus-Boelter thermal exchange coefficient (W/m²/K)

$\mathcal{H}_{\text{nb}}(t)$ = Forster-Zuber thermal exchange coefficient (W/m²/K).

r_{clad} = external radius of the clad (m).

The thermal exchange coefficient $\mathcal{H}_c(t)$ and $\mathcal{H}_{\text{nb}}(t)$ are function of the fluid temperature $T_{\text{fluid}}(t)$. Moreover, $\mathcal{H}_{\text{nb}}(t)$ is a function of $T_{\text{surf}}(t)$.

Substituting Eq. (1.43) into Eq. (2.1) for $\mathcal{K} = 1$, we obtain

$$r_{\text{clad}} \{ \mathcal{H}_{\text{nb}}(t) [T_{\text{surf}}(t) - T_{\text{sat}}(t)] + \mathcal{H}_c(t) [T_{\text{surf}}(t) - T_{\text{fluid}}(t)] \} = Z_1(t) - T_{\text{surf}}(t) Z_2(t). \quad (2.2)$$

From Eq. (2.2), we obtain

$$T_{\text{surf}}(t) = \frac{Z_1(t) + r_{\text{clad}} \mathcal{H}_{\text{nb}}(t) T_{\text{sat}}(t) + r_{\text{clad}} \mathcal{H}_c(t) T_{\text{fluid}}(t)}{Z_2(t) + r_{\text{clad}} \mathcal{H}_{\text{nb}}(t) + r_{\text{clad}} \mathcal{H}_c(t)} \quad (2.3)$$

and

$$\frac{q''_{\text{fluid}}(t)}{2\pi r_{\text{clad}}} = \frac{\mathcal{H}_{\text{nb}}(t) [Z_1(t) - Z_2(t) T_{\text{sat}}(t)] + \mathcal{H}_c(t) [Z_1(t) - Z_2(t) T_{\text{fluid}}(t)]}{Z_2(t) + r_{\text{clad}} \mathcal{H}_{\text{nb}}(t) + r_{\text{clad}} \mathcal{H}_c(t)} \quad (2.4)$$

so that

$$\frac{1}{2\pi r_{\text{clad}}} \frac{\partial q''_{\text{fluid}}}{\partial T_{\text{fluid}}}(t) = -\mathcal{H}_c(t) \left[\frac{Z_2(t)}{Z_2(t) + r_{\text{clad}} \mathcal{H}_{\text{nb}}(t) + r_{\text{clad}} \mathcal{H}_c(t)} \right]. \quad (2.5)$$

In steady-state cases, Eq. (1.44) can be used to simplify Eq. (2.3) as

$$T_{\text{surf}} = \frac{\varphi + \mathcal{H}_{\text{nb}} T_{\text{sat}} + \mathcal{H}_c T_{\text{fluid}}}{\mathcal{H}_{\text{nb}} + \mathcal{H}_c} \quad (2.6)$$

where the known heat flux delivered to coolant is $\varphi = q''_{\text{fluid}}/P_h$ with the heated perimeter defined as $P_h = 2\pi r_{\text{clad}}$.

If we write $h = h(T_{\text{fluid}}, p)$, the enthalpy of water, and consider a constant pressure process, we can write the following relation:

$$C_p(T, p) = \left(\frac{\partial h}{\partial T_{\text{fluid}}} \right)_p \quad (2.7)$$

where C_p is the specific heat capacity of water (J/K/kg) at constant pressure.

In this study, we are assuming single phase and forced convection flow regime. The mass flow equation in the coolant is based on the following balance relation

$$\frac{\partial}{\partial t} \rho(z, t) + \frac{\partial}{\partial z} Q(z, t) = 0 \quad (2.8)$$

where

$Q(z, t)$ = mass flow rate at an elevation z (kg/m²/s)

$\rho(z, t)$ = coolant density as given by the IAPWS-IF97 water tables as a function of temperature and pressure (kg/m³). May also be a function of the *flow regime*.

so that $Q(z)$ is constant in steady-state cases, with

$$Q(z, t) = V(z, t) \rho(z, t) \quad (2.9)$$

where $V(z, t)$ is the flow velocity (m/s).

The momentum conservation relation is responsible for the variation of the pressure along the channel as a function of mass flow rate gradient, gravity effect and coolant friction. This equation is written

$$\frac{\partial}{\partial t} Q(z, t) + \frac{\partial}{\partial z} \left[\frac{Q^2(z, t)}{\rho(z, t)} + p(z, t) \right] + \left[\frac{f Q^2(z, t)}{2\rho(z, t) D_H} + \rho(z, t) g \right] = 0 \quad (2.10)$$

where

$p(z, t)$ = pressure at an elevation z (Pa)

D_H = hydraulic diameter (m)

g = gravity acceleration constant (m/s²)

f = friction parameter as given by the Müller-Steinhagen correlation.

Note that the momentum conservation relation is currently not activated in module THM:. The pressure $p(z, t)$ is assumed constant along the channel.

The thermal convection equation in the coolant is based on a one-phase energy conservation relation in energy, written as

$$\frac{\partial}{\partial t} [h(z, t)\rho(z, t)] + \frac{\partial}{\partial z} [h(z, t)Q(z, t)] - \frac{1}{\mathcal{A}(z)} q''_{\text{fluid}}(z, t) = 0 \quad (2.11)$$

where

$q''_{\text{fluid}}(z, t)$ = linear power received by the fluid through clad (W/m)

$h(z, t)$ = coolant enthalpy (J/kg)

$\mathcal{A}(z)$ = cross section of the coolant channel (m²).

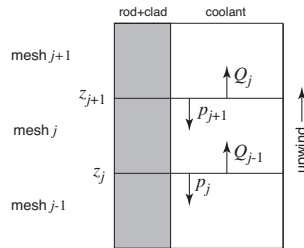


Figure 2: Definition of the axial mesh.

The method used to solve Eqs. (2.8) to (2.11) is based on a *finite volume discretization* along 1D axial meshes z_j , as depicted in Fig. 2. The main unknowns are the coolant flow rates Q_j , the clad surface $T_{\text{surf},j}$ and coolant $T_{\text{fluid},j}$ temperatures at each axial mesh, as a function of time. The spatial and time discretizations take into account the clad radius variation with axial mesh position z and time t . The cross section \mathcal{A}_j and height Δz_j of the channel section are assumed to be constant all over a mesh j . Equations (2.8) and (2.11) are solved in the *upwind direction* and Eq. (2.9) is solved in the *downwind direction*.

The integration of Eq. (2.8) over a mesh j delimited by z_j and z_{j+1} leads to

$$\frac{1}{\Delta t_n} \left[\int_{z_j}^{z_{j+1}} dz \rho \Big|_{t_n} - \int_{z_j}^{z_{j+1}} dz \rho \Big|_{t_{n-1}} \right] + [Q_j - Q_{j-1}] = 0. \quad (2.12)$$

The integration of Eq. (2.9) over a mesh j delimited by z_j and z_{j+1} leads to

$$\begin{aligned} \frac{1}{\Delta t_n} \left[\int_{z_j}^{z_{j+1}} dz Q \Big|_{t_n} - \int_{z_j}^{z_{j+1}} dz Q \Big|_{t_{n-1}} \right] + \left[\frac{Q_j^2}{\rho_j} - \frac{Q_{j-1}^2}{\rho_{j-1}} \right] + [p_{j+1} - p_j] \\ + \int_{z_j}^{z_{j+1}} dz \left[\frac{f Q^2}{2\rho D_H} + \rho g \right] = 0. \end{aligned} \quad (2.13)$$

The integration of Eq. (2.11) over a mesh j delimited by z_j and z_{j+1} leads to

$$\begin{aligned} \frac{1}{\Delta t_n} \left[\int_{z_j}^{z_{j+1}} dz h \rho \Big|_{t_n} - \int_{z_j}^{z_{j+1}} dz h \rho \Big|_{t_{n-1}} \right] + [h(z_{j+1})Q_j - h(z_j)Q_{j-1}] \\ - \frac{1}{\mathcal{A}_j} \int_{z_j}^{z_{j+1}} dz q''_{\text{fluid}} = 0 \end{aligned} \quad (2.14)$$

where the terms $h(z_j)$ are expressed as

$$h(z_j) = h(T_{\text{fluid},j-1}, p_{j-1}) \quad (2.15)$$

and the integrals in dz are expressed as

$$\int_{z_j}^{z_{j+1}} dz h \rho = [h(T_{\text{fluid},j-1}, p_{j-1}) \rho(T_{\text{fluid},j-1}, p_{j-1}) + h(T_{\text{fluid},j}, p_j) \rho(T_{\text{fluid},j}, p_j)] \frac{\Delta z_j}{2}. \quad (2.16)$$

Equation (2.14) can be rewritten as

$$\begin{aligned} h(T_{\text{fluid},j}, p_j) = \left[\frac{\rho(T_{\text{fluid},j}, p_j) \Delta z_j}{2\Delta t_n} \Big|_n + Q_j \right]^{-1} \\ \times \left[\left(-\frac{\rho(T_{\text{fluid},j-1}, p_{j-1}) \Delta z_j}{2\Delta t_n} \Big|_n + Q_{j-1} \right) h(T_{\text{fluid},j-1}, p_{j-1}) \right. \\ + \frac{\rho(T_{\text{fluid},j-1}, p_{j-1}) \Delta z_j}{2\Delta t_n} h(T_{\text{fluid},j-1}, p_{j-1}) \Big|_{n-1} \\ \left. + \frac{\rho(T_{\text{fluid},j}, p_j) \Delta z_j}{2\Delta t_n} h(T_{\text{fluid},j}, p_j) \Big|_{n-1} + \frac{1}{\mathcal{A}_j} \int_{z_j}^{z_{j+1}} dz q''_{\text{fluid}} \right] \end{aligned} \quad (2.17)$$

where q''_{fluid} is itself a function of $T_{\text{fluid},j}$. Equation (2.17) is therefore an implicit relation in $T_{\text{fluid},j}$ that can be solved by inverting the water function $h(T_{\text{fluid},j}, p_j)$.

In steady-state conditions, the mass flow rate $Q(z)$ is constant along the channel and Eq. (2.17) simplifies to an expression giving the increase in enthalpy over each axial mesh:

$$h_j = h_{j-1} + \frac{q''_{\text{fluid}}}{Q \mathcal{A}_j} \Delta z_j = h_{j-1} + \frac{P_h \varphi}{Q \mathcal{A}_j} \Delta z_j \quad (2.18)$$

where

P_h = heated perimeter (m)

φ = heat flux delivered to fluid (W/m²)

Q = mass flow rate (kg/m²/s)

\mathcal{A}_j = coolant cross section area (m²)

Δz_j = axial mesh width (m).

3 The flow regime model

The flow regime model of module THM: is based on legacy correlations function of flow characteristics such as the heat flux delivered to fluid, the fluid enthalpy, the clad surface temperature, and other thermodynamic properties of the fluid. Different flow regimes are identified along the coolant channel, as depicted in Fig. 3.

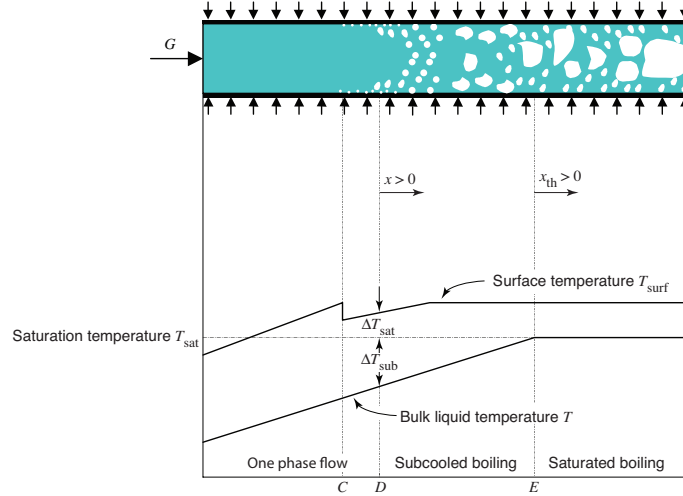


Figure 3: Surface and fluid bulk temperature distributions in subcooled and saturated boiling.

When a heated surface exceeds the saturation temperature of the surrounding fluid, boiling can occur. If the bulk fluid temperature is below the saturation temperature, boiling is referred to as *subcooled boiling* ($\mathcal{K} = 1$). If the bulk fluid temperature is equal to the saturation temperature, this is referred to as *saturated boiling* ($\mathcal{K} = 2$). *Thermodynamic equilibrium* between the liquid and the vapor phases can only exist in *saturated boiling* regime where both phases are at saturation temperature. If bulk fluid temperature is above the saturation temperature, the fluid is in *superheated steam* state ($\mathcal{K} = 3$).

The axial point C in the channel is the *onset of nuclear boiling* (ONB) where the wall temperature rises sufficiently above the local saturation temperature so that bubbles start to form at the heated wall. Point D is the *onset of fully developed boiling* (OFDB) where *bubble detachment* first occurs before the bulk liquid is saturated. It is assumed that contributions to steam voidage start at OFDB so that a *single-phase free convection* regime ($\mathcal{K} = 0$) can be set before this point.

The *thermodynamic quality* x_{th} is a characteristics of the boiling regime. It is obtained from the overall energy balance

$$x_{th} = \frac{\bar{h}(P, T_{cool}) - h_{sat,\ell}(P)}{h_{sat,v}(P) - h_{sat,\ell}(P)} \quad (3.1)$$

where

T_{cool} = bulk fluid temperature (K)

P = bulk fluid pressure (Pa)

\bar{h} = bulk fluid enthalpy (J/kg) as obtained from the solution of the Navier-Stokes equations

$h_{sat,\ell}$ = enthalpy of the saturated liquid at pressure P (J/kg)

$h_{sat,v}$ = enthalpy of the saturated vapor at pressure P (J/kg).

Since the thermodynamic quality is defined in terms of the fluid enthalpy, it can have values greater than one and less than zero. The thermodynamic quality is negative for subcooled boiling.

The water boiling model is based on the calculation of three non-dimensional flow quantities: the *flow quality* x , the *slip ratio* S and the *void fraction* α of the vapor. The *flow quality* x is the true flow fraction of the vapor phase and is always between zero and one, regardless of the fluid state. The thermodynamic quality corresponds to the flow quality only if thermodynamic equilibrium exists between the phases. We define the flow quality as

$$x = \frac{\dot{m}_v}{\dot{m}_v + \dot{m}_\ell} \quad (3.2)$$

where

\dot{m}_ℓ = mass flow rate of the liquid phase (kg/s)

\dot{m}_v = mass flow rate of the vapor phase (kg/s).

If the two phases are moving at the same speed, the corresponding *void fraction* is written

$$\alpha = \frac{\frac{x}{\rho_v}}{\frac{x}{\rho_v} + \frac{1-x}{\rho_\ell}} \quad (3.3)$$

where

ρ_ℓ = density of the saturated liquid at pressure P (kg/m³)

ρ_v = density of the saturated vapor at pressure P (kg/m³).

In order to characterize a two-phase flow, a slip ratio different than one is frequently introduced in addition of void fraction. The slip ratio is defined as the ratio of the average velocity of vapor phase flow u_v to the average velocity of liquid phase flow u_ℓ . The void fraction α can be related to the slip ratio S as follows:

$$S = \frac{u_v}{u_\ell} = \frac{\dot{m}x/\mathcal{A}\alpha\rho_v}{\dot{m}(1-x)/\mathcal{A}(1-\alpha)\rho_\ell} = \frac{\rho_\ell x(1-\alpha)}{\rho_v(1-x)\alpha} \quad (3.4)$$

where \mathcal{A} is the fluid cross-section area and $\dot{m} = \dot{m}_v + \dot{m}_\ell$ is the total mass flow rate (kg/s), so that the void fraction is now written

$$\alpha = \frac{\frac{x}{\rho_v}}{\frac{x}{\rho_v} + S \frac{1-x}{\rho_\ell}}. \quad (3.5)$$

Knowledge of the void fraction is required to compute the averaged density $\bar{\rho}$ of the saturated bulk fluid as

$$\bar{\rho} = \alpha\rho_v + (1-\alpha)\rho_\ell \quad (3.6)$$

which is next substituted in Eqs. (2.8) to (2.11).

The calculation of the flow quality in module THM: is based on a choice between two different models:

- The first model is an implementation of the Bowring model, as described in Sect. 4.1 of Mayinger text.^[2] According to this model, heat is removed from clad by two main mechanisms. The *agitation heat flux* φ_a is the convection mechanism on the boundary layer due to the temperature difference between clad surface and bulk fluid. The *evaporation heat flux* φ_e is related to bubble grows and is function of the latent heat of vaporization. The increase of flow quality is only due to the evaporation heat flux. Here, we are neglecting the heat flux due to single phase heat transfer between patches of bubbles.

The flow quality x_j in axial mesh j is approached using a relation similar to Eq. (2.18):

$$x_j = \begin{cases} 0 & \text{if } j \text{ is at OFDB;} \\ x_{j-1} + \frac{P_h \varphi_e}{Q \mathcal{A}_j (h_{\text{sat},v} - h_{\text{sat},\ell})} \Delta z_j & \text{otherwise} \end{cases} \quad (3.7)$$

with

$$\varphi_e = \frac{\varphi}{1 + \pi} \quad (3.8)$$

and

$$\pi = \frac{\varphi_a}{\varphi_e} = \frac{\rho_\ell C_\ell (T_{\text{surf}} - T_{\text{cool}})}{\rho_v (h_{\text{sat},v} - h_{\text{sat},\ell})} \quad (3.9)$$

where

P_h = heated perimeter (m)

φ = heat flux delivered to fluid (W/m²)

C_ℓ = specific heat capacity of the liquid (J/K/kg)

Q = mass flow rate (kg/m²/s)

\mathcal{A}_j = coolant cross section area (m²)

Δz_j = axial mesh width (m).

- The second option for computing the flow quality is a profile-fit model, as proposed in Sect. 5.2.2.2 of Ref. [3] and generally used with the Saha-Zuber correlation. The profile-fit model is based in the assumption that the main liquid enthalpy h_ℓ along the channel is a function of the bulk fluid enthalpy \bar{h} . The proposed approximation has the form^[4]

$$\frac{h_{\text{sat},\ell} - h_\ell}{h_{\text{sat},\ell} - h_{\ell,D}} = \exp\left(-\frac{\bar{h} - h_{\ell,D}}{h_{\text{sat},\ell} - h_{\ell,D}}\right) \quad (3.10)$$

where $h_{\ell,D}$ is the main liquid enthalpy on the OFDB point D .

The flow quality can be approximated by the following relation

$$x = \frac{\bar{h} - h_\ell}{h_{\text{sat},v} - h_\ell} \quad (3.11)$$

and is equal to the thermodynamic quality in the saturated region where $h_\ell = h_{\text{sat},\ell}$. Equation (3.10) gives $\bar{h} = h_\ell = h_{\ell,D}$ on point D , corresponding to a flow quality equal to zero. The liquid enthalpy $h_\ell \leq h_{\text{sat},\ell}$ is given from Eq. (3.11) as

$$h_\ell = \frac{\bar{h} - x h_{\text{sat},v}}{1 - x} \quad (3.12)$$

and can be used to obtain the liquid temperature in the subcooled region by inverting the water function $h_\ell(T_{\text{fluid}}, p)$.

Evaluation of Eq. (3.10) is based on the knowledge of the thermodynamic thermodynamic quality at OFDB, written as

$$x_{\text{th},D} = -\frac{h_{\text{sat},\ell} - h_{\ell,D}}{h_{\text{sat},v} - h_{\text{sat},\ell}} = -\frac{C_\ell \Delta T_{\text{sub},D}}{h_{\text{sat},v} - h_{\text{sat},\ell}} \quad (3.13)$$

where

C_ℓ = specific heat capacity of the liquid (J/K/kg)

$\Delta T_{\text{sub},D}$ = degree of subcooling ($T_{\text{sat}} - T_D$) of bulk fluid at OFDB (K).

We note that both x_{th} and $x_{\text{th},D}$ values are negative in the subcooled region. We combine Eqs. (3.10) and (3.11) to obtain

$$x = \frac{\bar{h} - h_{\text{sat},\ell} + (h_{\text{sat},\ell} - h_{\ell,D}) \exp\left(-\frac{\bar{h} - h_{\ell,D}}{h_{\text{sat},\ell} - h_{\ell,D}}\right)}{h_{\text{sat},v} - h_{\text{sat},\ell} + (h_{\text{sat},\ell} - h_{\ell,D}) \exp\left(-\frac{\bar{h} - h_{\ell,D}}{h_{\text{sat},\ell} - h_{\ell,D}}\right)}. \quad (3.14)$$

Substituting Eqs. (3.1) and (3.12) in Eq. (3.14), we obtain

$$x = \frac{x_{\text{th}} - x_{\text{th},D} \exp\left(\frac{x_{\text{th}}}{x_{\text{th},D}} - 1\right)}{1 - x_{\text{th},D} \exp\left(\frac{x_{\text{th}}}{x_{\text{th},D}} - 1\right)} \quad (3.15)$$

so that $x = 0$ if $\bar{h} = h_{\ell,D}$ and $x = 1$ if $x_{\text{th}} = 1$.

Accurate calculation of the slip ratio is important to obtain correct thermo-hydraulics quantities such as the coolant density. Assuming $S = 1$ leads to overprediction of the void fraction and to inexact value of the coolant density. The *drift flux model* is recommended for flow regimes typical to those found in pressurized or boiling water reactors.^[5] The velocity \bar{U}_{GU} (m/s) of the vapor relative to a reference frame moving with the coolant is given by a correlation of the form

$$\bar{U}_{GU} = 1.18 \left[\frac{\sigma_{\ell} g (\rho_{\ell} - \rho_{\text{v}})}{\rho_{\ell}^2} \right]^{1/4} \quad (3.16)$$

where

σ_{ℓ} = surface tension of the liquid (N/m)

g = acceleration due to gravity (9.81 m/s²)

ρ_{ℓ} = mass density of the liquid (kg/m³)

ρ_{v} = mass density of the vapor (kg/m³).

The drift flux model replaces Eq. (3.3) for the void fraction with the following expression:

$$\alpha = \frac{\frac{x}{\rho_{\text{v}}}}{C_0 \left(\frac{x}{\rho_{\text{v}}} + \frac{1-x}{\rho_{\ell}} \right) + \frac{\bar{U}_{GU}}{Q}} \quad (3.17)$$

where

C_0 = concentration parameter that corrects for the fact that the void concentration and velocity profiles across the channel can vary independently of one another. This parameter is taken as $C_0 = 1.13$.

Q = mass flow rate (kg/m²/s).

As depicted in Fig. 3, OFDB occurs at an axial location where bubble detachment first occurs before the bulk liquid is saturated. OFDB occurs if the bulk fluid enthalpy satisfies $\bar{h} \geq h_{\text{sat},\ell} - C_{\ell} \Delta T_{\text{sub},D}$. Different correlations are available to compute the degree of subcooling of bulk fluid at OFDB:

- Bowring correlates the temperature at which the bubbles detach by the simple equation^[6]

$$\Delta T_{\text{sub},D} = \frac{\eta \varphi}{v} \quad (3.18)$$

where

η = subcooled void parameter (K m³/J)

φ = heat flux delivered to fluid (W/m²)

v = liquid velocity (m/s)

and where the subcooled void parameter is a function of the fluid pressure P (Pa) written as

$$\eta = \frac{14 + 0.1 P}{1.01325 \times 10^5} \quad (3.19)$$

- The Saha-Zuber correlation is used to calculate the temperature subcooling at OFDB in the subcooled region.^[7] At low flow rates, the bubble detachment occurs at constant Nusselt number and is thermally controlled, while at high flow rate, the bubble departure is hydrodynamically induced at a fixed Stanton number. The Peclet number P_e is defined as

$$P_e = \frac{Q D_h C_\ell}{k_\ell} \quad (3.20)$$

where

Q = mass flow rate (kg/m²/s)

D_h = hydraulic diameter (m)

C_ℓ = specific heat capacity of the liquid (W/kg/K)

k_ℓ = thermal conductivity of the liquid (W/m/K).

The value $P_e = 70000$ is the limit between thermally and hydrodynamically controlled bubble departure. The Saha-Zuber correlation for the temperature subcooling at OFDB is written

$$\Delta T_{\text{sub},D} = \begin{cases} \frac{\varphi}{0.0065} \frac{Q C_\ell}{k_\ell} & \text{if } P_e > 70000; \\ \frac{\varphi D_h}{455} \frac{C_\ell}{k_\ell} & \text{if } P_e \leq 70000 \end{cases} \quad (3.21)$$

where φ is the heat flux delivered to fluid (W/m²).

Knowledge of the flow regime and flow quality is required to obtain the value of the heat transfer coefficients used in the Newton's law of cooling (2.1). These coefficients are function of the following non-dimensional numbers:

Nusselt number:

$$Nu = \frac{\mathcal{H} D_h}{k_\ell} \quad (3.22)$$

where

\mathcal{H} = heat transfer coefficient (W/m²/K)

D_h = hydraulic diameter (m)

k_ℓ = thermal conductivity of the liquid (W/m/K).

liquid Reynolds number:

$$Re_\ell = (1-x) \frac{Q D_h}{\mu_\ell} \quad (3.23)$$

where

Q = mass flow rate (kg/m²/s)

μ_ℓ = dynamic viscosity of the liquid (kg/m/s).

liquid Prandtl number:

$$Pr = \frac{\mu_\ell C_\ell}{k_\ell} \quad (3.24)$$

where

C_ℓ = specific heat capacity of the liquid (J/K/kg).

inverse Martinelli number:

$$X_{tt}^{-1} = \left(\frac{x}{1-x} \right)^{0.9} \left(\frac{\rho_\ell}{\rho_v} \right)^{0.5} \left(\frac{\mu_v}{\mu_\ell} \right)^{0.1} \quad (3.25)$$

where

- ρ_ℓ = mass density of the liquid (kg/m³)
 ρ_v = mass density of the vapor (kg/m³)
 μ_v = dynamic viscosity of the vapor (kg/m/s).

For fully developed (hydrodynamically and thermally) turbulent flow in a smooth circular tube, the local Nusselt number may be obtained from the Dittus-Boelter correlation:

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (3.26)$$

so that the Dittus-Boelter expression of the heat transfer coefficient is

$$\mathcal{H}_{sp} = 0.023 \frac{k_\ell}{D_h} Re^{0.8} Pr^{0.4} \quad (3.27)$$

The nucleate pool boiling correlation of Forster and Zuber is used to calculate the nucleate boiling heat transfer coefficient \mathcal{H}_{fz} and the turbulent flow correlation of Dittus-Boelter is used to calculate the liquid-phase convective heat transfer coefficient \mathcal{H}_c . The Forster and Zuber expression of the heat transfer coefficient is

$$\mathcal{H}_{fz} = 0.00122 \frac{k_\ell^{0.79} C_\ell^{0.45} \rho_\ell^{0.49}}{\mu_\ell^{0.29} \sigma_\ell^{0.5} (h_{sat,v} - h_{sat,l})^{0.24} \rho_v^{0.24}} (T_{surf} - T_{sat})^{0.24} [P_{sat}(T_{surf}) - P]^{0.75} \quad (3.28)$$

where

σ_ℓ = surface tension of the liquid (N/m)

P = bulk fluid pressure (Pa)

$P_{sat}(T_{surf})$ = saturation pressure as a function of temperature T_{surf} (Pa).

The Chen expression of the heat transfer coefficients are taking into account both the heat transfer coefficients due to nucleate boiling as well as forced convective regimes.^[8] They are written

$$\mathcal{H}_c = F \mathcal{H}_{sp} \quad \text{and} \quad \mathcal{H}_{nb} = S_{fz} \mathcal{H}_{fz} \quad (3.29)$$

where the two-phase multiplier F is a function of the inverse Martinelli number X_{tt}^{-1} written as

$$F = \begin{cases} 2.35 (0.213 + X_{tt}^{-1})^{0.736} & \text{if } X_{tt}^{-1} > 0.100207; \\ 0 & \text{else.} \end{cases} \quad (3.30)$$

The nucleate boiling suppression factor S_{fz} is the ratio of the effective superheat to wall superheat. It accounts for decreased boiling heat transfer because the effective superheat across the boundary layer is less than the superheat based on wall temperature. The suppression factor is written

$$S_{fz} = \frac{1}{1 + 2.53 \times 10^{-6} (Re_\ell F^{1.25})^{1.17}} \cdot \quad (3.31)$$

References

- [1] S. Nakamura, *Computational Methods in Engineering and Science*, John Wiley and Sons, New York (1977).
- [2] F. Mayinger, *Subcooled Boiling*, in *Two-Phase Flows and Heat Transfer with Application to Nuclear Reactor Design Problems*, J. J. Ginoux Editor, McGraw-Hill (1978).
- [3] R. T. Lahey and F. J. Moody, *The thermal-hydraulics of a boiling water nuclear reactor*, American Nuclear Society (1977).
- [4] N. Zuber, F. W. Staub and G. Bijwaard, "Vapor Void Fractions in Subcooled Boiling and Saturated Boiling Systems," *Proc. 3rd Int. Heat Transfer Conf.*, Chicago (1966).
- [5] N. Zuber and J. Findlay, "Average Volumetric Concentration in Two-Phase Systems," *Trans. ASME Jul. Ht. Transfer*, Vol 87, p 453 (1969).
- [6] R. W. Bowring, "Physical Model, Based on Bubble Detachment, and Calculation of Steam Voidage in the Subcooled Region of a Heated Channel," OECD Report HPR-10, 1962.
- [7] P. Saha and N. Zuber, "Point of net vapor generation and vapor void fraction in subcooled boiling," *Proc. of Fifth Int. Heat Transfer Conf.*, Vol. 4, 175–179 (1974).
- [8] W. Chen and X. Fang, "A note on the Chen correlation of saturated flow boiling heat transfer," *Int. J. of refrigeration*, **48**, 100–104 (2014).